Loxodrome (Mercator) course and distance on the WGS ellipsoid

The eccentricity ecc of the WGS84 ellipsoid is

ecc = 0.08181919084255

Latitude of the origin is ϕ_1 and longitude of the origin is λ_1

Latitude of the destination if ϕ_2 and longitude of the destination is λ_2

Course K is the direction, heading, from the origin to the destination

The approximate formula of course when all are in radians

If latitudes and longitudes are in radians, then

the approximate formula of course K from the origin to the destination is

$$K = \tan^{-1}\left\{ \frac{\lambda_2 - \lambda_1}{\ln\left(\dfrac{\tan(\frac{\pi}{4} + \frac{\phi_2}{2})}{\tan(\frac{\pi}{4} + \frac{\phi_1}{2})} \right) - a\cdot(\sin\phi_2 - \sin\phi_1) - b\cdot(\sin^3\phi_2 - \sin^3\phi_1) - c\cdot(\sin^5\phi_2 - \sin^5\phi_1)} \right\}$$

where $a = ecc^2 = 0.00669437999013$

$$b = \frac{ecc^4}{3} = 1.4938241150749E\text{-}5$$

$$c = \frac{ecc^6}{5} = 6.00013575883841E\text{-}8$$

Wthout terms a, b, c, the formula reduces to the spherical formula.

The approximate formula of course when longitude difference is in minutes

If you calculate $\lambda_2 - \lambda_1$ in minutes and latitudes are in radians, you get

$$K = \tan^{-1}\left\{\frac{\lambda_2 - \lambda_1}{3437.74677078 \cdot \ln\left(\frac{\tan(\frac{\pi}{4} + \frac{\phi_2}{2})}{\tan(\frac{\pi}{4} + \frac{\phi_1}{2})}\right) - A(\sin\phi_2 - \sin\phi_1) - B(\sin^3\phi_2 - \sin^3\phi_1) - C(\sin^5\phi_2 - \sin^5\phi_1)}\right\}$$

where A = 23.0135831934767

B = 0.051353890277194

C = 0.000206269473292

$\tan^{-1}$ is same as arcus tangent - the inverse tangent.

The formula of K is an approximation of the WGS ellipsoid formula.

Without terms A,B,C the formula reduces to the spherical formula.

Loxodrome distance

If latitude difference $\phi_2 - \phi_1$ is expressed in minutes, distance D in nautical miles is

$$D = \frac{\phi_2 - \phi_1}{\cos K}$$

Course and distance calculation using Meridional parts M(ϕ)

There are meridional parts in nautical tables.
We can define the meridional part M(ϕ) in minutes as

$$\mathrm{M}(\phi) = 3437.74677078494 \cdot \ln\left(\tan(\frac{\pi}{4}+\frac{\phi}{2})\right) - A\sin\phi - B\sin^3\phi - C\sin^5\phi$$

where ϕ is latitude.
Thus the meridional difference ΔM is

$$\Delta \mathrm{M} = \mathrm{M}(\phi_2) - M(\phi_1)$$

If longitude difference and meridional difference are expressed in minutes, course K is

$$\mathrm{K} = \tan^{-1}(\frac{\lambda_2 - \lambda_1}{\Delta M})$$

Loxodrome distance

If latitude difference $\phi_2 - \phi_1$ is expressed in minutes, distance D in nautical miles is

$$\mathrm{D} = \frac{\phi_2 - \phi_1}{\cos K}$$

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